**APRIL/MAY 2019**

# MMA15C — GRAPH THEORY

Time : Three hours

Maximum : 75 marks

SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL questions.

1. (a) Prove that a graph is bipartite if and only if it contains no odd cycle.

Or

(b) Prove that every connected graph contains a spanning tree.

2. (a) If $G$ is a block with $v \geq 3$, prove that any two edges of $G$ lie on a common cycle.

Or

(b) If $G$ is a Hamiltonian, prove that for every nonempty proper subset $S$ of $V$.

$\omega(G - S) \leq |S|$

3. (a) If $G$ is a $k$- regular bipartite graph with $k > 0$, prove that $G$ has a perfect matching.

Or

(b) Prove that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.

4. (a) Prove that a set $S \subseteq V$ is an independent set of $G$ if and only if $V \setminus S$ is a covering of $G$.

Or

(b) Prove that $r(k, l) \leq \binom{k+l-2}{k-1}$.

5. (a) Prove that a graph $G$ is embeddable in the plane if and only if it is embeddable on the sphere.

Or

(b) Prove that all planar embeddings of a given connected planar graph have the same number of faces.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. Prove that Let $T$ be a spanning tree of a connected graph $G$, and let $e$ be any edge of $T$. Then

(a) the cotree $\overline{T}$ contains no bond of $G$;

(b) $\overline{T} + e$ contains a unique bond of $G$.



7. Prove that $\kappa \leq \kappa' \leq \delta$.

8. Prove that a matching $M$ in $G$ is a maximum matching if and only if $G$ contains no $M$-augmenting path.

9. Prove that $r(k, k) \geq 2^{k/2}$.

10. If $G$ is a connected plane graph, Prove that

$\nu - \varepsilon + \phi = 2$.