3. (a) State and Prove Shifting property of z-transform.

Or

(b) Find $Z[\sin w_n]$

4. (a) Prove that Suppose that the matrix $A$ has $k$ linearly independent eigen-vectors $\xi_1, \xi_2, \ldots, \xi_k$ corresponding to $k$ eigen values $\lambda_1, \lambda_2, \ldots. \lambda_k$. If condition $\sum_{n=n_0}^{\infty} \frac{1}{|\lambda_1(n)|} \|B(n)\| < \infty$ holds for $B(n)$, then system equation $y(n+1) = [A + B(n)]y(n)$ has solutions $y_i(n), 1 \le i \le k$, such that $y_i(n) = \xi_i + o(1)]\lambda_i^n$.

Or

(b) Show that $\sin\left(n\pi + \frac{1}{n}\right) = 0\left(\frac{1}{n}\right) n \to \infty$

5. (a) Suppose that $f$ is continuous on $\mathbb{R}$ and satisfies the following assumptions:

(i) $xf(x) > 0, x \ne 0$

(ii) $\lim_{x\to 0} \inf \frac{f(x)}{x} = L, 0 < L < \infty$

(iii) $pL > \frac{k^k}{(k+1)^{k+1}}$ if $k \ge 1$ and $pL > 1$ if $k=0$, where $p = \lim_{n\to\infty} \inf p(n) > 0$.

Prove that for every solution of $x(n+1) - x(n) + p(n)f(x(n-k)) = 0$ oscillates.

Or

(b) Suppose that $p(n) \ge 0$ and $\sup p(n) < \frac{k^k}{k+1^{k+1}}$

Prove that $x(n+1) - x(n) + p(n)x(n-k) = 0$, $n \in \mathbb{Z}^+$ has a nonoscillatory solution.

SECTION B — $(3 \times 15 = 45$ marks)

Answer any THREE questions.

6. Consider the third-order difference equation

$$x(n+3) + 3x(n+2) - 4x(n+1) - 12x(n) = 0.$$

Show that the functions $2^n$, $(-2)^n$, and $(-3)^n$ form a fundamental set of solutions of the equation.

7. Solve the system $y(n+1) = A\,y(n) + g(n)$, where

$$A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}, g(n) = \begin{pmatrix} n \\ 1 \end{pmatrix}, y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

8. Solve the difference equation $x(n+2) + 3x(n+1) + 2x(n) = 0, x(0) = 1, x(1) = -4$

9. State and Prove the generalization of the Poincare — Perron theorem.

10. Suppose that $\lim_{n\to\infty} \inf p(n) = p > \frac{k^n}{(k-1)^{k+1.}}$

Prove that the following statements hold:

(a) $x(n+1) - x(n) + p(n)x(n-k) \leq 0,$

has no eventually positive solution

(b) $x(n+1) - x(n) + p(n)x(n-k) \geq 0,$

has no eventually negative solution

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NOVEMBER/DECEMBER 2019

MMA44 — DIFFERENCE EQUATION

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Prove that the operator $\Delta^{-1}$ is linear.

Or

(b) Verify that $\{n, 2^n\}$ is a fundamental set of solutions of the equation

$$x(n+2) - \frac{3n-2}{n-1}x(n+1) + \frac{2n}{n-1}x(n) = 0.$$

2. (a) Find a general solution of the system $x(n+1) = A\ x(n)$, where

$$A = \begin{pmatrix} 1 & -5 \\ 1 & -1 \end{pmatrix}$$

Or

(b) Find the solution of the difference system $x(n+1) = A\,x(n)$, where $A = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 2 & -4 \\ 0 & 1 & 6 \end{pmatrix}$