NOVEMBER/DECEMBER 2018

MMA24 — MECHANICS

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) State and prove the principle of conservation of energy.

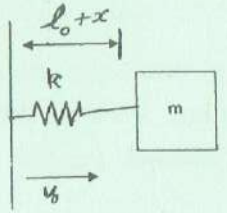
Or

(b) State and prove Konig's theorem.

2. (a) Find the differential equations of motion for a spherical pendulum of length l.

Or

(b) Suppose a mass-spring system is attached to a frame which is translating with a uniform velocity v_0 Let l_0 be the unstressed spring length and use the elongation x as the generalized Co ordinate.



Find the Jacobi integral for system.

3. (a) Find the curve joining two points along which a particle starting from rest and sliding down the curve without friction under the influence of a uniform gravitational field will reach the end of the curve in a minimum time.

Or

(b) State and prove Hamilton's principle

4. (a) State and prove Jacobi's theorem.

Or

(b) Derive the modified Hamilton-Jacobi equation.

5. (a) Show that the transformation $Q = \sqrt{2qe^t}\cos p$, $p = \sqrt{2qe^{-t}}\sin p$ is canonical.

Or

(b) If $u(q, p)$ and $v(q, p)$ are integrals of a Hamiltonian system then prove that the Poisson bracket $(u, v)$is a constant of the motion.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. (a) A particle of mass m is suspended by a massless wire of length $r = a + b\cos wt (a > b > 0)$ to form a spherical pendulum. Find the equation of motion.

(b) Prove that the angular momentum of a system of particles of total mass m about a fixed point O is equal to the angular momentum about O of a single particle of mass m which is moving with the centre of mass plus the angular momentum of the system about the centre of mass

7. Derive the standard form of Lagrange's equations for a holonomic system.

8. Derive Jacobi's form of the principle of least action.

9. Using the Hamilton Jacobi method solve the mass-spring problem.

10. Consider the transformation $Q = q - tp + \frac{1}{2}gt^2, P = p - gt$. Find K-H and the generating function.

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