NOVEMBER/DECEMBER 2018

MMA34 — PROBABILITY THEORY

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Let $\{A_n\}, n = 1, 2, \ldots,$ be a nonincreasing sequence of events and let A be their product. Prove that $P(A) = \lim_{n\to\infty} P(A_n)$.

Or

(b) Let $\{A_n\}, n = 1, 2, \ldots,$ be a nondecreasing sequence of events and let A be their alternative. Prove that $P(A) = \lim_{n\to\infty} P(A_n)$.

2. (a) Prove that the expected value of the product of an arbitrary finite number of independent random variables, whose expected values exist, equals the product of the expected values of these variables.

Or

(b) Prove that the coefficient of correlation satisfies the double inequality. $-1 \le \rho \le 1$.

3. (a) Find the characteristic function and the moments of normal distribution.

Or

(b) Find the density function of the random variable X, whose characteristic function is

$$\phi_1(t) = \begin{cases} 1-|t| & \text{for } |t| < 1, \\ 0 & \text{for } |t| > 1 \end{cases}$$

4. (a) Test whether the addition theorem is valid for random variables with gamma distributions.

Or

(b) Obtain the characteristic function and its moments of uniform distribution.

5. (a) Let $F_n(x) = (n = 1,2,...)$ be the distribution function of the random variable X_n. Prove that the sequence $\{X_n\}$ is stochastically convergent to zero if and only if the sequence $\{F_n(x)\}$ satisfies the relation.

$$\lim_{n\to\infty} F_n(x) = \begin{cases} 0 & \text{for } x \le 0, \\ 1 & \text{for } x > 0. \end{cases}$$

Or

(b) State and prove De Moivre Laplace theorem.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. State and prove Baye's theorem.

7. If a random variable Y can take on only non-negative values and has expected value $E(Y)$. Prove that for an arbitrary positive number K.

$$P(Y \ge K) \le \frac{E(Y)}{K}.$$

8. Let $F(x,y)$, $F_1(x), F_2(y), \phi(t,u), \phi_1(t)$, and $\phi_2(u)$ denote the distribution functions and the characteristics functions of the random variables $(X,Y), X$ and y, respectively. Prove that the random variable X and Y are then independent if and only if the equation $\phi(t,u) = \phi_1(t)\phi_2(u)$ holds for all real t and u.

9. Let the random variable X_n have a binomial distribution defined by the formula

$P(X_n = r) = \dfrac{n!}{r!(n-r)!} p^r (1-\rho)^{n-r}$, where r takes on the values $0,1,2,...,n$. If for $n = 1,2,...$ the relation $p = \dfrac{\lambda}{n}$ holds where $\lambda > 0$ is a constant, prove that $\lim_{n\to\infty} P(X_n = r) = \dfrac{X}{r!} e^{-\lambda}$.

10. State and prove Levy – Cramer theorem.