NOVEMBER/DECEMBER 2018

MMA14 — DIFFERENTIAL GEOMETRY

Time : Three hours | Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Find the length of the curve given as the intersection of the surfaces $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $x = a\cosh(z/a)$ from the point $(a, 0, 0)$ to the point (x, y, z).

Or

(b) Obtain the curvature and torsion of a curve given as the intersection of two surfaces.

2. (a) A helicoid is generated by the screw motion of a straight line which meets the axis at an angle α. Find the orthogonal trajectories of the generators.

Or

(b) Find the surface of revolution which is isometric with a region of the right helicoid.

3. (a) Show that every point P of a surface has a neighborhood which is convex and simple.

Or

(b) Derive the normal property of geodesics.

4. (a) State and prove Gauss-Bonnet theorem.

Or

(b) Calculate the circumference of a geodesic circle of small radius r.

5. (a) Prove that the edge of regression of the rectifying developable has equation $R = r + k\dfrac{(\tau t + kb)}{k^{-1}\tau - k\tau'}$.

Or

(b) Prove that at any point P on a surface there is a paraboloid such that the normal curvature of the surface at P in any direction is the same as that of the pataboloid.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. Obtain the curvature and torsion of the curve of intersection of the two quadratic surfaces. $ax^2 + by^2 + cz^2 = 1$, $a^1 x^2 + b^1 y^2 + c^1 z^2 = 1$.

7. Find the coefficient of the direction which makes an angle $1/2\,\pi$ with the direction whose coefficients are (l, m).

8. Prove that, on the general surface a necessary and sufficient condition that the curve $v = c$ be geodesic is $EE_2 + FE_1 - 2EF_1 = 0$. When $v = c$, for all values of u.

9. Derive the Liouville's formula for K_g.

10. Prove that the necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.