APRIL/MAY 2019

MMA43 — MATHEMATICAL STATISTICS

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) The random variable $X_k (k = 1,2,3,\ldots 16)$ are independent and have the same density $f(x) = \frac{1}{2\sqrt{\pi}} e^{-\left[-\frac{1}{2}x\frac{(x-2)^2}{4}\right]}$, find the distribution function and also find $P(0 \leq X \leq 2)$.

Or

(b) The sequence $\{F_n(t)\}$ of distribution function of student's t with n degrees of freedom satisfies for every t, prove that $Lt_{n\to\alpha} F_n(t) = \frac{1}{2\sqrt{\pi}} \int_{-\alpha}^{t} e^{-t^2/2} dt$.

2. (a) Explain the Wald - Wolfowitz and Wilcoxon - Mann-Whitney tests.

Or

(b) Discuss about independence test by contingency table.

(b) Prove that a family F is normal if and only if its closure F^- with respect to the distance function $\rho(f, g) = \sum_{k=1}^{\infty} \delta_k(f, g) 2^{-k}$ is compact.

3. (a) Prove that a continuous function $u(z)$ which satisfies condition

$$u(z_0) = \frac{1}{2\pi} \int_0^{2\pi} u(z_0 + re^{i\theta})\, d\theta$$

is necessarily harmonic.

Or

(b) Describe the Harnack's principle.

4. (a) Show that an elliptic function without poles is constant.

Or

(b) Prove that the zeros $a_1, \ldots, a_n$ and poles $b_1, \ldots, b_n$ of an elliptic function satisfy $a_1 + \cdots + a_n \equiv b_1 + \cdots + b_n (\operatorname{mod} M)$.

5. (a) Prove that the modular function $\lambda(r)$ effects a one-to-one conformal mapping of the region Ω onto the upper half plane. The mapping extends continuously to the boundary in such a way that $\tau = 0, 1, \infty$ correspond to $\lambda = 1, \infty, 0$.

Or

(b) Show that any elliptic function with periods ω_1, ω_2 can be written as $C \prod_{k=1}^{n} \frac{\sigma(z - a_k)}{\sigma(z - b_k)}$ (C = const.)

SECTION B — ($3 \times 15 = 45$ marks)

Answer any THREE questions.

6. Derive Jensen's formula.

7. Prove that $\zeta(s) = 2^s \pi^{s-1} \sin \frac{\pi s}{2} \Gamma(1-s) \zeta(1-s)$.

8. Prove that given any simply connected region Ω which is not the whole plane, and a point $z_0 \in \Omega$, there exists a unique analytic function $f(z)$ in Ω, normalized by the conditions $f(z_0) = 0$, $f'(z_0) > 0$, such that $f(z)$ defines a one-to-one mapping of Ω onto the disk $|w| < 1$.

9. Prove that there exists a basis (ω_1, ω_2) such that the ratio $\tau = \omega_2 / \omega_1$ satisfies the following conditions : (a) $\operatorname{Im} \tau > 0$, (b) $-\frac{1}{2} < \operatorname{Re} \tau \leq \frac{1}{2}$, (c) $|\tau| \geq 1$, (d) $\operatorname{Re} \tau \geq 0$ if $|\tau| = 1$. The ratio τ is uniquely determined by these conditions, and there is a choice of two, four, or six corresponding bases.

10. Derive the Legendre's relation.