NOVEMBER/DECEMBER 2019

MMA43 — MATHEMATICAL STATISTICS

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Derive the distribution of the arithmetic mean of independent normally distributed random variables.

Or

(b) From a population in which the characteristic X the normal distribution N(1;2). Draw a simple of size n = 12, observe the following values of X

X_1	X_2	X_3	X_4	X_5	X_6
2.0	2.5	0.5	1.0	0.0	–0.9
X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
5.1	–1.5	0.8	1.1	0.8	0.4

What is the probability that Z will exceed or equal the value obtained z = 32.28?

2. (a) Explain the Wald-Wolfowitz and Wilcoxon-Mann-Whitney tests.

Or

(b) Explain the test of the Kolmogrov and Smirnov type.

3. (a) Let the distribution function F(x) depend upon one parameter Q that is m = 1. If there exists a sufficient estimate U of the parameter Q. prove that the solution of equation $\frac{\partial \log L}{\partial \lambda_1} = 0$ is a function of U only.

Or

(b) Explain the methods of finding estimates.

4. (a) Explain about one way classification.

Or

(b) Discuss about unbiased test.

5. (a) Obtain the OC function of SPRT.

Or

(b) Explain the testing a hypothesis concerning the parameter p of a zero-one distribution.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. The random variables X_k(k=1,2,....,8) are independent and have the same normal distribution N(0;2). Consider statistic $\chi^2 = \sum_{k=1}^{8} X_k^2$, the random variable χ^2 has eight degrees of freedom find the expected value and the standard deviation of this random variable.

7. Let $S_{1n_1}(x)$ and $S_{2n_2}(x)$ be two empirical distribution functions of two independent simple samples drawn from the same population, in which the characteristics X has a continuous distribution function prove that

$$Lt_{\substack{n_1 \to \infty \\ n_2 \to \infty}} Q_{n_1 n_2}(\lambda) = \begin{cases} \sum_{j=-\infty}^{\infty} (-1)^j e^{-2j^2\lambda^2}, & for\,\lambda > 0 \\ 0, & for\,\lambda \le 0 \end{cases}$$

8. Let V be an unbiased estimate of the parameter Q and let U be a sufficient estimate of Q. Prove that the random variable E(V | u) is an unbiased estimate of Q. If moreover the variance $D^2(V)$ exists the inequality $D^2[E)V \mid u)] \le D^2(V)$ holds. $D^2[E(V \mid u)] = D^2(v)$ holds only if $E(V \mid u) = V$. with probability one.

9. State and Prove Fisher Lemma.

10. State and Prove auxiliary theorem.