9. Assume that $\sum_{n=0}^{\infty} an$ converges absolutely and has sum A and suppose $\sum_{n=0}^{\infty} bn$ converges with sum B. Prove that the Cauchy product of these two series converges and has sum AB.

10. let α be of bounded variation on $[a, b]$. Assume that each term of the sequence $\{fn\}$ is real valued function such that $fn \in R(\alpha)$ on $[a, b]$ for each $n=1,2,\ldots$ Assume that $fn \to f$ uniformly on $[a,b]$ and define $gn(x) = \int_a^x fn(t)\, d(\alpha(t))$ if $x \in [a,b]$, n=1,2... Prove that the following

(a) $f \in R(\alpha)\ on\ [a,b]$

(b) $gn \to g$ uniformly on $[a,b]$, where $g(x) = \int_a^x f(t)\, d\,\alpha(t)$.

NOVEMBER/DECEMBER 2019

MMA12 — REAL ANALYSIS – I

Time : Three hours | Maximum : 75 marks

SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL questions.

1. (a) If f is bounded on $[a,b]$, prove that f is of bounded variation on $[a,b]$.

Or

(b) State and prove additive property of total variation.

2. (a) Assume that $\alpha \nearrow$ on $[a,b]$. If $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a,b]$. and if $f(x) \le g(x)$ for all x in $[a,b]$, prove that $\int_a^b f(x) d\alpha(x) \le \int_a^b g(x) d\alpha(x)$.

Or

(b) If $f \in R(\alpha)$ and if $g \in R(\alpha)$ on $[a,b]$, prove that $c_1 f + c_2 g \in R(\alpha)$ on $[a,b]$ (for any two constants c_1 and c_2 and $\int_a^b (c_1 f + c_2 g) d\alpha = c_1 \int_a^b f d\alpha + c_2 \int_a^b g d\alpha$.

3. (a) State and prove second mean value theorem for Riemann – Stieljes integrals.

Or

(b) Let α be of bounded variation on $[a,b]$ and assume that $f \in R(\alpha)$ and $[a,b]$. Prove that $f \in R(\alpha)$ on every subinterval $[c,d]$ of $[a,b]$.

4. (a) State and prove Abel's test.

Or

(b) If a series is convergent with sum s, prove that it is also $(c,1)$ summable with cesaro sum s.

5. (a) Assume that $fn \to f$ uniformly on s, If each fn is continous at a point c of s, prove that the limit function f is also continous at c.

Or

(b) Assume that $\lim_{n \to \infty} fn = f$ on $[a,b]$. If $g \in R$ on $[a,b]$, define $h|x| = \int_a^x f|t|g|t|\,dt$., $h_n|x| = \int_a^x fn|t|g|t|\,dt$ if $x \in [a,b]$, prove that $hn \to h$ uniformly on $[a,b]$.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. Let f be of bounded variation on $[a, b]$. If $x \in [a, b]$, let $V(x) = V_f(a,x)$ and put $V(a) = 0$. Prove that every point of continuity of f is also a point of continuity of v and the converse is also true.

7. Assume that $\alpha \nearrow$ on $[a, b]$. Prove that the following statements are equivalent.

(a) $f \in R(\alpha)$ on $[a,b]$

(b) f satisfies Riemann's conditions with respect to α on $[a, b]$.

(c) $\underline{I}(f,\alpha) = \overline{I}(f,\alpha)$.

8. Let $Q = \{(x,y) : a \le x \le b,\ c \le y \le d\}$. Assume that α is of bounded variation on $[a, b]$, β is of bounded variation on $[c, d]$ and f is continuous on Q. If $(x,y) \in Q$ define $F(y) = \int_a^b f(x,.y)\,d\alpha(x)$, $G(x) = \int_c^d f(x,y)\,d\beta(y)$. Prove that $F \in R(\beta)$ on $[c,d]$, $G \in R(\alpha)$ on $[a,b]]$ and $\int_c^d f(y)\,d\beta(y) = \int_a^b G(x)\,d\alpha(x)$.