NOVEMBER/DECEMBER 2019

MMA23 — PARTIAL DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL questions.

1. (a) Find the general integral of partial differential equation $y^2 p - xyq = x(z - 2y)$.

Or

(b) Using Lagrange's method, solve the equation

$$\begin{vmatrix} x & y & z \\ \alpha & \beta & y \\ \dfrac{\partial z}{\partial x} & \dfrac{\partial z}{\partial y} & -1 \end{vmatrix} = 0, \text{ where } z = z(x, y).$$

2. (a) Derive the canonical form for hyperbolic equation.

Or

(b) Find the characteristics of the equation $u_{xx} + 2u_{xy} + \sin^2 x\, u_{yy} + u_y = 0$ when its is of hyperbolic type.

3. (a) Obtain the solution of Laplace equation in a cylindrical co-ordinates.

Or

(b) Derive Poisson equation.

4. (a) Obtain the solution of diffusion equation in spherical co-ordinates.

Or

(b) In an one dimensional infinite solid, $-\infty < x < \infty$, the surface $a<x<b$ is initially maintained at temperature T_0 and at zero temperature every where outside the surface. Prove that

$$T(x,t)=\frac{T_0}{2}\left[erf\left(\frac{b\ \ x}{\sqrt{4\alpha t}}\right)\ \ erf\left(\frac{a\ \ x}{\sqrt{4\alpha t}}\right)\right]$$

where erf is a error function.

5. (a) Obtain the periodic solution of the wave equation in the form $u(x,t)=Ae^{i(kx\pm wt)}$, where $i=\sqrt{-1}$, $k=\pm\frac{w}{c}$ A is a constant

Or

(b) Derive the D' Alembert's solution of the initial value problem

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. Show that the partial differential equations $xp-yq=x$ and $x^2p+q=xz$ are compatible and find their solution.

7. Obtain the canonical from for elliptic equation.

8. Find the solution of Laplace equation in cylindrical co-ordinates.

9. A uniform rod of length L whose surface is thermally insulated is initially at temperature $\theta=\theta_0$. At the time $t=0$ one end is suddenly cooled to $\theta=0$ and subsequently maintained at this temperature, the other end remains thermally insulated. Find the temperature distribution $\theta=(x,t)$.

10. A rectangular membrane with fastened edges makes free transverse vibrations. Explain how a formal series solution can be found.