NOVEMBER/DECEMBER 2019

MMA11 — ALGEBRA – I

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Prove that $N|a|$ is a subgroup of G.

Or

(b) Show that the conjungacy is an equivalence relation on G.

2. (a) Let G be a group and suppose that G is the internal direct product of $N_1, N_2, \ldots N_n$. Let $T = N_1 \times N_2 \times \ldots \times N_n$. Prove that G and T are isomorphic.

Or

(b) Show that any finite abelian group is the direct product.

3. (a) Show that the two nilpotent linear transformation are similar if and only if they have the same invariants.

Or

(b) If $T \in A(V)$ has all its characteristic roots in F. Prove that there is a basis of V in which the matrix of T is triangular.

4. (a) Prove that every linear transformation $T \in A_F(V)$ satisfies its characteristic polynomial.

Or

(b) Show that for each $i = 1, 2, \ldots k$, $v_i \neq 0$ and $v = v_1 \oplus v_2 \oplus \ldots + V_k$. The minimal polynomial of T_i is $q_i(x)^{l_i}$.

5. (a) If A is invertible, prove that $(AC\ A^{-1}) = t_r c$.

Or

(b) For $A, B \in F_n$ and $\lambda \in F$.,

Prove that

(i) $t_r(\lambda A) = \lambda t_r(A)$

(ii) $t_r(A+B) = t_r(A) + t_r(B)$

(iii) $t_r(AB) = t_r(BA)$

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. If P is a prime number and $P/_{O(G)}$, Prove that G has an element of order P.

7. Prove that every finite abelian group is the direct product of cyclic groups.

8. Show that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$.

9. Prove that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.

10. Show that $T \geq 0$. If and only if $T = AA*$ for some A.