APRIL/MAY 2018

MMA24 — MECHANICS

Time : Three hours Maximum : 75 marks

SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL questions.

1. (a) State and prove the principle of virtual work.

Or

(b) A particle of mass m is suspended by a massless wire of length $r = a + b\cos\omega t$ $(a > b > 0)$ to form a spherical pendulum. Find the equation of motion.

2. (a) A double pendulum consists of two particles suspended by massless rods. Assuming that all motion takes place in a vertical plane, find the differential equation of motion.

Or

(b) Derive the Jacobi integral.

3. (a) Find the stationary values of the function $f = z$ subject to the constraints

$$\phi_1 = x^2 + y^2 + z^2 - 4$$
$$\phi_2 = xy - 1$$

Or

(b) Derive Hamiltons canonical equation of motion.

4. (a) Derive the Hamilton Jacobi equation

Or

(b) Solve the mass-spring system by Hamilton-Jacobi method.

5. (a) Show that the transformation $Q = \frac{1}{2}(q^2 + p^2)$

$P = -\tan^{-1}\frac{q}{p}$ is canonical.

Or

(b) Discuss the momentum transformation.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. With the usual notations, prove that the rotational kinetic energy of a rigid body can be expressed in the form $T_{rot} = \frac{1}{2}\omega^T I \omega$.

7. Derive the standard form of Lagrange's equations for a holonomic system.

8. State and prove that principle of least action. Deduce its Jacobi's form.

9. (a) Derive the Hamilton-Jacobi equation for conservative system.

(b) Derive the Hamilton-Jacobi equation for a conservative system with ignorable co-ordinates.

10. Obtain the four major types of generating function associated with this transformation.

$$Q = \log\frac{\sin p}{q} \qquad P = q\cot p$$

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