APRIL/MAY 2018

MMA14 — DIFFERENTIAL GEOMETRY

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Show that $[\dot{r}, \ddot{r}, \dddot{r}] = 0$ is necessary and sufficient that the curve be plane,

Or

(b) Prove that the necessary and sufficient condition that a curve be straight line is $k = 0$.

2. (a) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes z = constant.

Or

(b) Derive the double family of curves.

3. (a) Prove that the curves of the family $v^3/u^2 =$ constant are geodesics on a surface with metric

$$v^2du^2 - 2uvdudv + 2u^2dv^2 (u > 0, v > 0).$$

Or

(b) Derive the canonical geodesic equations.

4. (a) Prove that if the orthogonal trajections of the curves $v =$ constant are geodesics then H^2/E is independent of u.

Or

(b) Prove that the geodesic curvature vector of any curve is orthogonal to the curve.

5. (a) Obtain the Rudrigue's formula.

Or

(b) Prove that the necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. Obtain the curvature and torsion of a curve given as the intersection of two surfaces.

7. A helicoid is generated by the screw motion of a straight line which meets the axis at an angle α. Find the orthogonal trajectories of the generation. Find also the metric of the surface referred to the generators and their orthogonal trajectories as parametric curves.

8. If $g(t)$ is continuous for $0 < t < 1$ and if $\int_0^1 v(t)g(t)dt = 0$ for all admissible function $v(t)$, prove that $g(t) = 0$.

9. Let P be a point of a given curve C on a surface and Q the point of C at a distance δs from P along C. Let $\overline{C}$ be the geodesic are PQ, of length $\delta \overline{s}$. Prove that if δQ is the angle between C and $\overline{C}$ at P and if $\delta \phi$ is the angle between $\overline{C}$ and C at Q, the geodesic curvature of C at P is $\lim_{\delta s \to 0} \frac{\delta\theta + \delta\phi}{\delta s}$.

10. Derive the second fundamental form.