APRIL/MAY 2018

# MMA21 — ALGEBRA II

Time : Three hours Maximum : 75 marks

## SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL the questions.

1. (a) If L is a finite extension of $K$ and if $K$ is a finite extension of $F$ prove that $L$ is a finite extension of $F$.

Or

(b) Prove that the elements in $K$ which are algebraic over $F$ form a subfield of $K$.

2. (a) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

Or

(b) Prove that the polynomial $f(x) \in F(x)$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non-trivial common factor.

3. (a) If $K$ is a field and if $\sigma_1, \sigma_2 \ldots\ldots \sigma_n$ are distinct automorphism of K then prove that it is impossible to find $a_1, a_2, \ldots\ldots a_n$, not all 0, in K such that $a_1\sigma_1(u) + a_2\sigma_2(u) + \ldots\ldots + a_n\sigma_n(u) = 0$ for all $u \in K$.

Or

(b) Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F

4. (a) If $p(x) \in F(x)$ is solvable by radicals over $F$ prove that the Galois group over $F$ of $p(x)$ is a solvable group.

Or

(b) Let $G$ be a finite abelian group enjoying the property that the relation, $x^n = e$ is satisfied by atmost $n$ elements of $G$, for every integer $n$. Prove that $G$ is a cyclic group.

5. (a) Let $C$ be the field of complex numbers and suppose that the division ring $D$ is algebraic over $C$. Prove that $D = C$.

Or

(b) (i) For all $x, y \in Q$, prove that $N(xy) = N(x)N(y)$

(ii) If $a \in H$, prove that $a^{-1} \in H$ if and only if $N(a) = 1$



SECTION B — ($3 \times 15 = 45$ marks)

Answer any THREE questions.

6. Prove that the element $a \in K$ is algebraic over $F$ if and only if $F(a)$ is a finite extension of $F$.

7. If $F$ is of characteristic zero and if $a$, $b$ are algebraic over $F$ then prove there exists an element $c \in F(a, b)$ such that $F(a, b) = F(c)$

8. Let K be a normal extension of F and et H be a subgroup of $G(K, F)$; let $K_H = \{x \in K | \sigma(x) = x$ for all $\sigma \in H\}$ be the fixed field of $H$. Prove that

(a) $[K : K_H] = o(H)$

(b) $H = G(K, K_H)$.

9. Prove that a finite division ring is necessarily a commutative field.

10. State and prove left-Division Algorithm.