NOVEMBER/DECEMBER 2019

MMA41 — COMPLEX ANALYSIS – II

Time : Three hours Maximum : 75 marks

SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL questions.

All questions carry equal marks.

1. (a) Prove that an entire function of fractional order assume every finite value infinitly many times.

Or

(b) Prove a necessary and sufficient condition that absolute converence of the produce $\prod_{1}^{\infty}(1+a_n)$ converges to the series $\sum_{1}^{\infty}|a_n|$.

2. (a) Prove that the ζ –function can be extended to a meromorphic function in the whole plane whose only pole is a simple pole at s = 1 with the residue 1.

Or

(b) Prove that a family $\mathfrak{F}$ of analytic functions is normal with respect to $\mathcal{C}$ if and only if the function $\mathfrak{F}$ are uniformly bounded on every compact set.

3. (a) Derive Harnack's inequality.

Or

(b) Describe Mapping on a rectangle.

4. (a) Prove that a nonconstant elliptic function has equally many poles as it has zeros.

Or

(b) Derive the Fourier Development.

5. (a) Prove that $\mathfrak{P}''(z) = -\dfrac{\sigma(2z)}{\sigma(z)^4}$.

Or

(b) Prove that every $\mathcal{T}$ in the upper half plane is equivalent under the congruence subgroup *mod 2* to exactly one point in $\overline{\Omega} \cup \Omega$.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. Prove that the genus and the order of an entire function satisfy the double inequality $h \leq \lambda \leq h+1$.

7. State and prove Arzela's theorem.

8. State and prove Schwarz-Christoffel formula.

9. Show that any two bases of the same module are connected by a unimodular transformation.

10. Derive the Weierstrass function.