NOVEMBER/DECEMBER 2018

MMA13— ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL questions.

1. (a) Solve : $y'' + 2iy' + y = 0$.

Or

(b) Solve : $y'' + (3i-1)y' - 3iy = 0$.

2. (a) State and prove uniqueness theorem.

Or

(b) Solve : $y''' - 5y'' + 6y' = 0$.

3. (a) Let $\phi_1, \phi_2, \ldots, \phi_n$ be n solutions of $L(y) = 0$ on I satisfying $\phi_i^{(i-1)}(x_0) = 1, \phi_i^{(j-1)}(x_0) = 0, \quad j \neq i$. If ϕ is any solution of $L(y) = 0$ on I, Prove that there are n constant $c_1, c_2, \ldots, c_n$, such that $\phi = c_1\phi_1 + \ldots + c_n\phi_n$.

Or

(b) On solution of $x^2y'' - 2y = 0$, on $0 < x < \infty$ is $\phi_1(x) = x^2$. Find all solution of $x^2y'' - 2y = 2x - 1$ on $0 < x < \infty$.

452

4. (a) Solve : $2x^2y'' + xy' - y = 0$.

Or

(b) Compute the indicial polynomials and their roots for the equation $x^2y'' + (x + x^2)y' - y = 0$.

5. (a) Find the real – valued solution of the equation $y' = \dfrac{x + x^2}{y + y^2}$.

Or

(b) Prove that a function ϕ is a solution of the initial value problem $y' = f(x,y)$, $y(x_0) = y_0$ on an interval I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t,y)dt$ on I.

SECTION B — ($3 \times 15 = 45$ marks)

Answer any THREE questions.

All questions carry equal marks.

6. Let ϕ_1, ϕ_2 be two solutions of $L(y) = 0$ on an interval I and let x_0 be any point in I. Prove that ϕ_1, ϕ_2 are linearly independent on I if and only if $W(\phi_1, \phi_2)(x_0) \neq 0$.

7. Consider the equation with constant coefficients $L(y) = p(x)e^{ax}$, where p is the polynomial given by $p(x) = b_0x^m + b_1x^{m-1} + \ldots + b_m$ $(b_0 \neq 0)$.

Suppose 'a' is a root of the characteristic polynomial p of L of multiplicity j. Prove that there is a unique solutions ψ of $L(y) = p(x)e^{ax}$ of the form $\psi(x) = x^j(c_0x^m + c_1x^{m-1} + \ldots + c_m)e^{ax}$. Where $c_0, c_1, \ldots c_n$ are constants determined by the annihilator method.

8. Let $\phi_1, \phi_2, \ldots, \phi_n$ be n solution of $L(y) = 0$ on an interval I, and let x_0 be any point in I, prove that $w(\phi_1, \phi_2, \ldots, \phi_n)(x) =$

$$\exp\left[-\int_{x_0}^{x} a_1(t)dt\right] w(\phi_1, \phi_2, \ldots, \phi_n)(x_0).$$

9. Derive the Bessel function of order α of the first kind.

10. Prove that the existence theorem for convergence of the successive approximation.