APRIL/MAY 2018

MMA13 — ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours | Maximum : 75 marks

SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL questions.

1. (a) Solve $y'' - 4y = 0$.

Or

(b) Solve $y'' - 4y' + 5y = 0$.

2. (a) State and prove existence theorem.

Or

(b) Solve $y''' - 8y = 0$.

3. (a) Prove that there exists n linearly independent solutions of $L(y) = 0$ on I.

Or

(b) Reduce the order of a homogeneous equation.

4. (a) Solve $x^2 y'' + 2xy' - 6y = 0$.

Or

(b) Find the singular points of the following equation $x^2 y'' + (x + x^2) y - y = 0$ and determine those which are regular singular point.

5. (a) Find the real-valued solutions of the equations $y' = x^2 y$.

Or

(b) Explain the method of successive approximation.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. Show that two solutions ϕ_1, ϕ_2 of $L(y) = 0$ are linearly independent on an interval I if, and only if, $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I.

7. Let ϕ be any solution of $L(y) = y^{(n)} + a_1 y^{(n-1)} + ... + a_n y = 0$ on an interval I containing a point x_0. Prove that for all x in I.

$\|\phi(x_0)\| e^{-k|x-x_0|} \leqq \|\phi(x)\| \leqq \|\phi(x_0)\| e^{k|x-x_0|}$
where $K = 1 + |a_1| + ... + |a_n|$.

8. Prove that if $\phi_1, \phi_2, ..., \phi_n$ are n solutions of $L(y) = 0$ on an interval I, they are linearly independent there if, and only if $W\|\phi_1, \phi_2, ... \phi_n\|(x) \neq 0$ for all x in I.

9. Derive the Bessel function.

10. Let M, N be two real valued functions which have continuous first partial derivatives on some rectangle $R : |x - x_0| \leqq a$, $|y - y_0| \leqq b$ prove that the equation $M(x, y) + N(x, y) y' = 0$ is exact in R if and only if, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.