APRIL/MAY 2019

MMA34— PROBABILITY THEORY

Time : Three hours Maximum : 75 marks

SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL questions.

1. (a) Let $\{An\}$, $n = 1, 2, ...$ be a non decreasing sequence of events and let A be their alternative, prove that $P(A) = \lim_{n \to \infty} P(An)$.

Or

(b) Obtain the necessary and sufficient condition for the independence of the random variables X and Y.

2. (a) State and prove addition theorem of mathematical expectations.

Or

(b) Prove that the equality $\rho^2 = 1$ is a necessary and sufficient condition for the relation $P(y = ax + b) = 1$.

3. (a) Find the characteristic function of uniform distribution.

Or

(b) Prove that difference of two independent Poisson random variable is not a Poisson variate.

4. (a) Find the mean and variance of Beta distribution.

Or

(b) State and prove addition theorem of Cauchy distribution.

5. (a) State and prove the Levy-Cramer second theorem.

Or

(b) The random variables X_n $(n = 1, 2, ...)$ are independent and each of them has the Poisson distribution $P(x_n = r) = \frac{2^r}{r!} e^{-2}$ $(r = 0, 1, 2, ...)$. Find the probability that the sum $Y_{100} = X_1 + X_2 + ... + X_{100}$ is greater that 190 and less than 210.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. If the events $A_1, A_2, ...$ satisfy the assumptions of absolute probability and $P(B) > 0$, prove that for $i = 1, 2, ...$

$$P(Ai/B) = \frac{P(Ai)\, P(B/Ai)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + ...}.$$

7. State and prove Chebyshev inequality.

8. The joint distribution of the random variable (X, Y) is given the density

$$f(x, y) = \begin{cases} 1/4\,(1 + xy)\,(x^2 - y^2), & for\ |x| \le 1, |y| \le 1 \\ 0, & \text{otherwise} \end{cases}$$

show that the random variable X and Y are dependent.

9. Prove that the limiting case of binomial distribution is Poisson distribution.

10. State and prove De Moivre – Laplace theorem.