NOVEMBER/DECEMBER 2018

MMA23 — PARTIAL DIFFERENTIAL EQUATIONS

Time : Three hours

Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Find the integral surface of the linear partial differential equation $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+y=0, z=1$.

Or

(b) Find the characteristics of the equation $pq = z$ and hence determine the integral surface which passes through the parabola $x=0$ $y^2=z$.

2. (a) Reduce the following equation to a canonical form

$$(1+x^2)u_{xx} + (1+y^2)u_{yy} + xu_x + yu_y = 0.$$

Or

(b) If $L_u = a(x)\frac{d^2u}{dx^2} + b(x)\frac{du}{dx} + c(x)u$ then find its adjoint L^* Construct an adjoint to $L(u) = u_{xx} + u_{yy}$.

3. (a) Obtain the solution of Laplace equation in cylindrical co-ordinates.

Or

(b) Find a general spherically symmetric solution of the Helmholtz equation $(\nabla^2 - k^2)u = 0$.

4. (a) A one-dimensional infinite region, $-\infty < x < \infty$ is initially kept at zero temperature. A heat source of strength g_s units, situated at $x = \varepsilon$ releases heat instantaneously at time $t = \tau$. Determine the temperature in the region for $t > \tau$.

Or

(b) Solve by separation of variables $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$.

5. (a) Obtain the D'Alemberts solution of the one dimensional wave equation.

Or

(b) State and prove uniqueness theorem.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. (a) Show that the equations $xp = yq$ $z(xp + yq) = 2xy$ are compatible and solve them.

(b) Find the complete integral of $(p^2 + q^2)y = qz$ using Charpits method.

7. Explain: Riemann's method.

8. Explain: Dirichlet problem for a rectangle.

9. A uniform rod of length L whose surface is thermally insulated is initially at temperature $\theta = \theta_0$. At time $t = 0$, one end is suddenly cooled out at $\theta = 0$ and subsequently maintained at this temperature; the other end remains thermally insulated. Find the temperature distribution $\theta(x,t)$.

10. Obtain the solution of one-dimensional wave equation by canonical reduction.