APRIL/MAY 2019

MMA24 — MECHANICS

Time : Three hours   Maximum : 75 marks

SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL questions.

1. (a) Prove that the total kinetic energy is the sum of three parts.

   (i) The kinetic energy due to a particle having a mass m and moving with reference point P

   (ii) The kinetic energy of the system due to its motion relative to P and

   (iii) The scalar product of the velocity of the reference point and the linear momentum of the system relative to the reference point.

   Or

   (b) A particle of mass m is suspended by a massless wire of length

   $r = a + b\cos\omega t (a > b > 0)$

   to form a spherical pendulum. Find the equation of motion.

2. (a) Derive the expression for the kinetic energy of a system of N particles interms of the n generalized co ordinates $q_1, q_2, ...q_n$.

Or

(b) Two particles of mass m_1 and m_2 are connected by a light sting of length l which passes over a smooth pulley. Obtain the equation of motion.

3. (a) Derive the Hamilton canonical equations of motion.

Or

(b) A particle of mass is attracted to a fixed point O by an inverse square law that is $F_r = -m\mu / r^2$ where μ is the gravitational coefficient. Using the plane polar co ordinates, find the equation of motion.

4. (a) Obtain the Hamilton Jacobi equation.

Or

(b) State and prove Jacobi's theorem.

5. (a) Show that the transformation $Q = \sqrt{e^{-2q} - p^2}$, $Q = \cos^{-1}(pe^q)$ is canonical, using Poisson bracket.

Or

(b) Explain momentum transformation.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. (a) Derive Lagrangian form of d'Alembert's principle. (8)

(b) State and prove that principle of virtual work. (7)

7. Derive the standard form of Lagrange's equations for a holonomic system.

8. Find the path of minimum length between two given points on the two dimensional surface of a sphere of radius r.

9. State and prove Stackel 's theorem.

10. Obtain the tour major types of generating function associated with this transformation

$$Q = \log \frac{\sin p}{q} \quad P = q \cot p.$$