APRIL/MAY 2018

MMA15C — GRAPH THEORY

Time : Three hours

Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Prove that $\sum_{v \in V} d(v) = 2e$.

Or

(b) Let T be a spanning tree of a connected graph G and let e be an edge of G not in T. Prove that $T + e$ contains a unique cycle.

2. (a) If G is Hamiltonian, prove that for every non-empty proper subset S of V, $\omega(G - S) \leq |S|$.

Or

(b) Prove that $C(G)$ is well defined.

3. (a) Let M be a matching and K be a covering such that $|M| = |K|$. Prove that M is a maximum matching and K is a minimum covering.

Or

(b) Let G be a connected graph that is not an odd cycle. Prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.

4. (a) Prove that $r(K,l) \leq \binom{K+l-2}{K-1}$.

Or

(b) Show that every K – chromatic graph has atleast K vertices of degree atleast $K-1$.

5. (a) Let υ be a vertex of a planar graph G. Prove that G can be embedded in the plane in such a way that υ is on the exterior face of the embedding.

Or

(b) If G is a simple planar graph with $\upsilon \geq 3$, prove that $e \leq 3\upsilon - 6$.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. Let T be a spanning tree of a connected graph G, and let e be any edge of T. Prove that (a) the cotree $\overline{T}$ contains no bond of G. (b) $\overline{T} + e$ contains a unique bond of G.

7. Prove that $K \leq K' \leq \delta$.

8. Show that a matching M in G is a maximum matching if and only if G contains no M – augmenting path.

9. State and prove Ramsey's theorem.

10. Prove that every planar graph is 5-vertex-colourable.

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