8. State and prove Implicit function theorem,

9. (a) If f is a bounded measurable function on $[a,b]$ prove that $f \in L[a,b]$.

   (b) If $E \subset [a,b]$ prove that $\overline{m}E + \underline{m}E' = b - a$ where $E' = [a,b] - E$.

10. Prove that the metric space $L^2[a,b]$ is complete.

APRIL/MAY 2018

MMA22 — REAL ANALYSIS – II

Time : Three hours Maximum : 75 marks

SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL the questions.

1. (a) Let $\{\phi_0, \phi_1, \phi_2, ...\}$ be orthonormal and I and assume that $f \in L^2(I)$. Define two sequences of functions $\{s_n\}$ and $\{t_n\}$ on I as follows.

$$s_n(x) = \sum_{k=0}^{n} c_k \phi_k(x) \quad t_n(x) = \sum_{k=0}^{n} b_k \phi_k(x) \quad \text{where}$$

$c_k = (f, \phi_k)$ for $k = 0,1,2$ and $b_0, b_1, b_2 ...$ are arbitrary complex numbers. The prove that , for each n $\|f - s_n\| \le \|f - t_n\|$.

Or

   (b) State and prove Weierstrass Approximation theorem.

2. (a) Assume that g is differentiable at which total derivative $g'(a)$. Let $b = g(a)$ and assume that f is differentiable at b with total derivative $f'(b)$ prove that the composite function $h = f \circ g$ is differentiable at a.

Or

   (b) State and prove Mean-value theorem.

3. (a) Let $B = B(a; r)$ be an $n-$ ball in R^n, let ∂B denotes its boundary $\partial B = \{x : \|x - a\| = r\}$ and let $\overline{B} = B \cup \partial B$ denote its closure. Let $f = (f_1, \ldots f_n)$ be continuous on $\overline{B}$, and assume that all the partial derivatives $D_j f_i(x)$ exists if $x \in B$. Assume further that $f(x) \neq f(a)$, if $x \in \partial B$ and that the Jacobian determinant $J_f(x) \neq 0$ for each x in B. Prove that $f(B)$. the image of B and under f, contains as n-ball with centre at $f(a)$

Or

(b) Assume that $f = (f_1, \ldots f_n)$ has continuous partial derivatives $D_j f_i$ on an open set in R^n and that the Jacobian determinant $J_f(a) \neq 0$ for some point a in S. Prove that there is an n-ball $B(a)$ on which f is one –to-one.

4. (a) If G_1, G_2,.... are open subsets of $[a, b]$ prove that $\left|\bigcup_{n=1}^{\infty} G_n\right| \leq \sum_{n=1}^{\infty} |G_n|$.

Or

(b) If $E_1, E_2, \ldots$ are measurable subsets of $[a, b]$ and if $E_1 \subset E_2 \subset E_3 \ldots$ prove that $\bigcup_{n=1}^{\infty} E_n$ is measurable and $m \left(\bigcup_{n=1}^{\infty} E_n\right) = \lim_{n \to \infty} mEn$.

5. (a) Let f and g be not negative-value function on $[a, b]$. If $f, g \in L[a, b]$ prove that $f, g \in L[a, b]$.

Or

(b) State and prove the Schwarz inequality.

SECTION B — ($3 \times 15 = 45$ marks)

Answer any THREE questions.

6. (a) Assume that $f \in L(I)$. Prove that for each real β $\lim_{\alpha \to \infty} \int_1 f(t) \sin(\alpha t + \beta) dt = 0$.

(b) If g is of bounded variation on $[0, \delta]$ prove that $\lim_{\alpha \to +\infty} \frac{2}{\pi} \int_0^{\delta} g(t) \frac{\sin \alpha t}{t} dt = g(0+)$.

7. (a) Let f and $D_2 f$ continuous on a rectangle $[a, b] \times [c, d]$. Let p and q be differentiable on $[c, d]$ where $p(y) \in [a, b]$ and $q(y) \in [a, b]$ for each y in $[c, d]$. Define F by the equation.

$F(y) = \int_{p(y)}^{q(y)} f(x, y) dx$ if $y \in [c, d]$. Prove that $F'(y)$ exists for each in $[c, d]$ is given by $F(y) = \int_{p(y)}^{q(y)} D_2 f(x\ y) dx + f(q(y), y)$

$q'(y) - f(p(y), y) p'(y)$.

(b) If f is differentiable at C then prove that f is continuous at C.