8. Obtain the inverse Z-transform of $\bar{x}(z) = \dfrac{z(z+1)}{(z-1)^2}$.

9. State and prove the generalization of the Poincare-Perron theorem.

10. Consider the Pielou logistic delay equation

$y(n+1) = \dfrac{\alpha\, y(n)}{1+\beta\, y(n-k)}$, $\alpha > 1$, $\beta > 0$, k a positive integer. Show that every positive solution of above oscillates about its positive equilibrium point $y^* = (\alpha - 1)/\beta$ if $\dfrac{\alpha - 1}{\alpha} > \dfrac{K^k}{(k+1)^{k+1}}$.

APRIL/MAY 2019

MMA44 — DIFFERENCE EQUATION

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Prove that for fixed $k \in \mathbb{Z}^+$ and $x \in \mathbb{R}$, the following

(i) $\Delta x^{(k)} = k x^{(k-1)}$;

(ii) $\Delta^n x^{(k)} = k(k-1),\ldots,(k-n+1)\, x^{(k-n)}$;

(iii) $\Delta^k x^{(k)} = k!$.

Or

(b) Prove that the operator Δ^{-1} is linear.

2. (a) Find the solution of the difference system $x(n+1) = Ax(n)$, where $A = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 2 & -4 \\ 0 & 1 & 6 \end{pmatrix}$.

Or

(b) State and prove Abel's formula.

3. (a) Find the Z-transform of the sequences $\{na^n\}$ and $\{n^2a^n\}$.

Or

(b) State and prove shifting property.

4. (a) Show that $\left(\frac{n}{t^2+n^2}\right)^n = 0\left(\frac{1}{t^n}\right)$, $n \to \infty$, for $n \in \mathbb{Z}^+$.

Or

(b) Prove that suppose that the matrix A has k linearly independent eigen-vectors $\varepsilon_1, \varepsilon_2, ..., \varepsilon_k$ and k corresponding eigen values $\lambda_1, \lambda_2, ..., \lambda_k$. If condition $\sum_{n=n_0}^{\infty} \frac{1}{|\lambda_i(n)|} \| B(n) \| < \infty$ holds for $B(n)$, then system equation $y(n+1) = [A + B(n)]y(n)$ has solution $y_i(n), 1 \le i \le k$, such that $y_i(n) = [\xi_i + o(1)]\lambda_i^n$.

5. (a) Suppose that f is continuous on $\mathbb{R}$ and satisfies the following assumptions :

(i) $xf(x) > 0, x \neq 0$

(ii) $\lim_{x\to 0} \inf \frac{f(x)}{x} = L, 0 < L < \infty$,

(iii) $pL > \frac{K^k}{(k+1)^{k+1}}$ if $k \ge 1$ and $pL > 1$ if $k = 0$, where $p = \lim_{n\to\infty} \inf p(n) > 0$. Prove that for every solution of $x(n+1) - x(n) + p(n)f(x(n-k)) = 0$ oscillates.

Or

(b) Suppose that $p(n) \ge 0$ and sup $p(n) < \frac{k^K}{(k+1)^{k+1}}$. Prove that $x(n+1) - x(n) + p(n)x(n-k) = 0$, $n \in \mathbb{Z}^+$ has a nonoscillatory solution.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. Solve the equation
$x(n+3) - 7x(n+1) + 16x(n+1) - 12x(n) = 0$,
$x(0) = 0, x(1) = 1, x(2) = 1$.

7. Solve the system $y(n-1) = Ay(n) + g(n)$, where

$A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$, $g(n) = \begin{pmatrix} n \\ 1 \end{pmatrix}$, $y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.