NOVEMBER/DECEMBER 2018

MMA32 — TOPOLOGY

Time : Three hours Maximum : 75 marks

SECTION A — $(5 \times 6 = 30$ marks)

Answer ALL questions.

1. (a) Prove that the topologies of $\mathbb{R}_{\ell}$ and $\mathbb{R}_{K}$ are strictly finer than the standard topology $\mathbb{R}$,but not comparable with one another.

Or

(b) Let Y be a subspace of X. prove that a set A is closed in Y. Iff it equals the intersection of a closed set of X with Y.

2. (a) State and prove pasting lemma.

Or

(b) State and prove sequence lemma.

3. (a) Prove that a finite Cartesian product of connected space is connected.

Or

(b) Show that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

4. (a) State and prove extreme value theorem.

Or

(b) Let X be locally compact Hausdorff. Let A be a subspace of X. If A is closed in X or open in X, prove that A is locally compact.

5. (a) Prove that every metrizable space is normal.

Or

(b) Show that every compact Hausdorff space is normal.

SECTION B — ($3 \times 15 = 45$ marks)

Answer any THREE questions.

6. Let A be a subset of the topological space X.

(a) Prove that $x \in \overline{A}$ iff over open set U containing x intersects A.

(b) Supposing the topology of X is given by a basis, prove that $x \in \overline{A}$ iff every basis element B containing x intersects A.

7. Prove that the topologies on $\mathbb{R}^n$ induced by the euclidean metric d and the square metric ρ and the same as the product topology on $\mathbb{R}^n$.

8. Let L is the linear continuum in the order topology, prove that L is connected and so are intervals are rays in L.

9. State and prove tube lemma.

10. State and prove Uryshon lemma.

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