

NOVEMBER/DECEMBER 2018

MMA13— ORDINARY DIFFERENTIAL
EQUATIONS

Time : Three hours

Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Solve : $y'' + 2iy' + y = 0$.

Or

(b) Solve : $y'' + (3i - 1)y' - 3iy = 0$.

2. (a) State and prove uniqueness theorem.

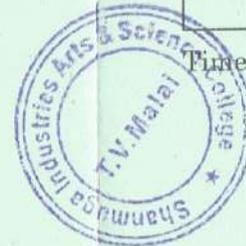
Or

(b) Solve : $y''' - 5y'' + 6y' = 0$.

3. (a) Let $\phi_1, \phi_2, \dots, \phi_n$ be n solutions of $L(y) = 0$ on I satisfying $\phi_i^{(i-1)}(x_0) = 1, \phi_i^{(j-1)}(x_0) = 0, j \neq i$. If ϕ is any solution of $L(y) = 0$ on I , Prove that there are n constant c_1, c_2, \dots, c_n , such that $\phi = c_1\phi_1 + \dots + c_n\phi_n$.

Or

(b) On solution of $x^2y'' - 2y = 0$, on $0 < x < \infty$ is $\phi_1(x) = x^2$. Find all solution of $x^2y'' - 2y = 2x - 1$ on $0 < x < \infty$.



4. (a) Solve : $2x^2y'' + xy' - y = 0$.

Or

(b) Compute the indicial polynomials and their roots for the equation $x^2y'' + (x + x^2)y' - y = 0$.

5. (a) Find the real - valued solution of the equation $y' = \frac{x + x^2}{y + y^2}$.

Or

(b) Prove that a function ϕ is a solution of the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ on an interval I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_0}^x f(t, y) dt$ on I.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

All questions carry equal marks.

6. Let ϕ_1, ϕ_2 be two solutions of $L(y) = 0$ on an interval I and let x_0 be any point in I. Prove that ϕ_1, ϕ_2 are linearly independent on I if and only if $W(\phi_1, \phi_2)(x_0) \neq 0$.

7. Consider the equation with constant coefficients $L(y) = p(x)e^{ax}$, where p is the polynomial given by $p(x) = b_0x^m + b_1x^{m-1} + \dots + b_m$ ($b_0 \neq 0$).

Suppose 'a' is a root of the characteristic polynomial p of L of multiplicity j . Prove that there is a unique solutions ψ of $L(y) = p(x)e^{ax}$ of the form $\psi(x) = x^j(c_0x^m + c_1x^{m-1} + \dots + c_m)e^{ax}$. Where c_0, c_1, \dots, c_m are constants determined by the annihilator method.

8. Let $\phi_1, \phi_2, \dots, \phi_n$ be n solution of $L(y) = 0$ on an interval I, and let x_0 be any point in I, prove that $W(\phi_1, \phi_2, \dots, \phi_n)(x) =$

$$\exp\left[-\int_{x_0}^x a_1(t) dt\right] W(\phi_1, \phi_2, \dots, \phi_n)$$

(x_0) .

9. Derive the Bessel function of order α of the first kind.
10. Prove that the existence theorem for convergence of the successive approximation.

