NOVEMBER/DECEMBER 2018

MMA31 — COMPLEX ANALYSIS - I

Time : Three hours

Maximum : 75 marks

SECTION A — ($5 \times 6 = 30$ marks)

Answer ALL questions.

1. (a) Prove that an analytic function in a region Ω whose derivative vanishes identically must reduce to a constant.

Or

(b) Prove that line integral $\int_{\gamma} p\,dx + q\,dy$ defined in Ω depends on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with partial derivatives $\dfrac{\partial U}{\partial x} = p$, $\dfrac{\partial U}{\partial y} = q$.

2. (a) State and prove Cauchy's integral formula.

Or

(b) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

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3. (a) If $p\,dx + q\,dy$ is locally exact in Ω then prove that $\int_\gamma p\,dx + q\,dy = 0$ for every cycle $\gamma \sim 0$ in Ω.

Or

(b) If $f(z)$ is analytic except for isolated singularities a_j in a region Ω then prove that $\frac{1}{2\pi i}\int_\gamma f(z)\,dz = \sum_j n(\gamma, a_j)$ $\mathrm{Res}_{z=aj} f(z)$ for any cycle γ which is homologous to zero in Ω and does not pass through any of the points a_j.

4. (a) Evaluate $\int_0^\pi \frac{d\theta}{a + \cos\theta}, a > 1$.

Or

(b) If u_1 and u_2 are harmonic in a region Ω then prove that $\int_\gamma u_1 * du_2 - u_2 * du_1 = 0$

5. (a) If $f_n(z)$ is analytic in the region Ω_n and that the sequence $\{f_n(z)\}$ converges to a limit function $f(z)$ in a region Ω, uniformly on every compact subset of Ω then prove that $f(z)$ is analytic in Ω.

Or

(b) State and prove that Hurwitz theorem

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. State and prove Cauchy's theorem for a rectangle.

7. If $\phi(\zeta)$ is continuous on the arc γ then prove that the function $F_n(Z) = \int_\gamma \frac{\phi(\zeta)}{(\zeta - z)^n} d\zeta$ is analytic in each of the region determined by γ and its derivative is $F_n'(Z) = nF_{n+1}(Z)$.

8. State and prove Rouche's theorem

9. Derive Poisson's formula

10. State and prove Schwarz's theorem.

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