8. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

9. State and prove Wedderburn theorem.

10. State and prove Frobenius theorem.

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NOVEMBER/DECEMBER 2018

MMA21 — ALGEBRA – II

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL the questions.

1. (a) If L is an algebraic extension of K and if K is an algebraic extension of F then prove that L is an algebraic extension of F.

Or

(b) If a and b in K are algebraic over F of degree m and n respectively then prove that $a \pm b$, ab and $a|b$ (if $b \neq 0$) are algebraic over F of degree atmost mn.

2. (a) If $p(x)$ is a polynomial in $F[x]$ of degree $n \geq 1$ and is irreducible over F then prove that there is an extension E of F such that $[E:F] = n$ in which $p(x)$ has a root.

Or

(b) If $f(x) \in F[x]$ is irreducible then prove that

(i) If the characteristic of F is 0, $f(x)$ has no multiple roots

(ii) If the characteristic of F is $p \neq 0$, $f(x)$ has a multiple root only if it is of the form $f(x) = g(x^p)$.

3. (a) Let F be a field and let $F(x_1,...,x_n)$ be the field of rational functions in $x_1,...x_n$ over F. Suppose that S is the field of symmetric rational function then prove that.

(i) $[F(x_1,...x_n) : S] = n!$

(ii) $G(F(x_1,...x_n), S) = S_n$ the symmetric group of degree n.

Or

(b) Prove that

(i) The fixed field of G is a sub-field of K

(ii) If K is the field of complex numbers and F is the field of real numbers then find $G(K,F)$.

4. (a) Prove that F is solvable if and only if $G^{(k)} = (e)$ for some integer k.

Or

(b) If F is a finite field $\alpha \neq 0, \beta \neq 0$ are two elements of F then prove that, we can find elements a and b such that $1 + \alpha a^2 + \beta b^2 = 0$.

5. (a) Prove that the adjoint in Q satisfies

(i) $x^{**} = x$

(ii) $(\delta x + \gamma y)^* = \delta x^* + \gamma y^*$.

(iii) $(xy)^* = y^* x^*$ for all x, y in Q and for all real δ and γ.

Or

(b) Let C be the field of complex numbers and suppose that the division ring D is 'algebraic over C. Then prove that $D = C$.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. Prove that the number e is transcendental.

7. If F is of characteristic zero and if a, b are algebraic over F then prove there exists an element $c \in F(a,b)$ such that $F(a,b) = F(c)$.