9. A conducting bar of uniform cross-section lies along the x-axis with ends at $x = 0$ and $x = L$. It is kept initially at temperature $0°$ and its lateral surface is insulated. There are no heat sources in the bar. Then end $x = 0$ is kept at $0°$ and heat is suddenly applied at the end $x = L$, so that there is a constant flux q_0 at $x = L$. Find the temperature distribution in the bar for $t > 0$.

10. Explain : Boundary and initial value problems for two dimensional equation method of eigen function.

APRIL/MAY 2019

MMA23 — PARTIAL DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Find the equation of the integral surface of the differential equation

$$2y(z-3)p+(2x-z)q=y(2x-3)$$

which passes through the circle $z=0$, $x^2+y^2=2x$.

Or

(b) Find the characteristic of the equation $pq=xy$ and determine the integral surface which passes through the curve $z=x$, $y=0$.

2. (a) Reduce the following equation to a canonical form and hence solve it

$$yu_{xx}+(x+y)u_{xy}+xu_{yy}=0.$$

Or

(b) If $L_u=a(x)\dfrac{d^2u}{dx^2}+b(x)\dfrac{du}{dx}+c(x)u$ then find its adjoint L^*.

3. (a) Explain the Neumann problem for a rectangle.

Or

(b) Obtain the solution of Laplace equation in spherical coordinates.

4. (a) In a one-directional infinite solid, $-\infty < x < \infty$, the surface $a < x < b$ is initially maintained at temperature T_0 and at zero temperature everywhere outside the surface. Show that

$$T(x, t) = \frac{T_0}{2}\left[erf\left(\frac{b-x}{\sqrt{4\alpha t}}\right) - erf\left(\frac{a-x}{\sqrt{4\alpha k}}\right)\right]$$

where erf is an error function.

Or

(b) If $f(t)$ is any continuous function prove that $\int_{-\infty}^{\infty} \delta(t-a) f(t)\, dt = f(a)$. If $\delta(t)$ is a continuously differentiable Dirac delta function vanishing for large t prove that $\int_{-\infty}^{\infty} f(t)\delta'(t)\, dt = -f'(0)$.

5. (a) Obtain the periodic solution of the wave equation in the form $u(x, t) = Ae^{i(kx \pm wt)}$ where $i = \sqrt{-1}$ $k = \pm w/c$, A is constant and hence define various terms involved in wave propagation.

Or

(b) State and prove uniqueness theorem of the solution for the wave equation.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. (a) Find the complete integral of the equation $(p^2 + q^2)x = pz$.

(b) Show that the equation $xp - yq = x$, $x^2p + q = xz$ are compatible and find their solution.

7. Explain the Riemann's method.

8. A homogeneous thermally conducting cylinder occupies the region $0 \le r \le a$, $0 \le \theta \le 2\pi$, $0 \le z \le h$ where r, θ, z are cylindrical coordinates. The top $z = h$ and the lateral surface $r = a$ are held at 0°, while base $z = 0$ is held at 100°. Assuming that there are no sources of heat generation within the cylinder, find the steady-temperature distribution within the cylinder.