

3. (a) State and Prove Shifting property of z-transform.

Or

- (b) Find  $Z[\sin \omega_n]$

4. (a) Prove that Suppose that the matrix  $A$  has  $k$  linearly independent eigen-vectors  $\xi_1, \xi_2, \dots, \xi_k$  corresponding to  $k$  eigen values

$$\lambda_1, \lambda_2, \dots, \lambda_k. \text{ If condition } \sum_{n=n_0}^{\infty} \frac{1}{|\lambda_1(n)|} \|B(n)\| < \infty$$

holds for  $B(n)$ , then system equation  $y(n+1) = [A + B(n)]y(n)$  has solutions  $y_i(n), 1 \leq i \leq k$ , such that  $y_i(n) = \xi_i + o(1)\lambda_i^n$ .

Or

- (b) Show that  $\sin\left(n\pi + \frac{1}{n}\right) = O\left(\frac{1}{n}\right) \rightarrow 0$

5. (a) Suppose that  $f$  is continuous on  $\mathbb{R}$  and satisfies the following assumptions:

(i)  $xf(x) > 0, x \neq 0$

(ii)  $\lim_{x \rightarrow 0} \inf \frac{f(x)}{x} = L, 0 < L < \infty$

(iii)  $pL > \frac{k^k}{(k+1)^{k+1}}$  if  $k \geq 1$  and  $pL > 1$  if

$$k=0, \text{ where } p = \lim_{n \rightarrow \infty} \inf p(n) > 0.$$

Prove that for every solution of  $x(n+1) - x(n) + p(n)f(x(n-k)) = 0$  oscillates.

Or

- (b) Suppose that  $p(n) \geq 0$  and  $\sup p(n) < \frac{k^k}{k+1^{k+1}}$

Prove that  $x(n+1) - x(n) + p(n)x(n-k) = 0, n \in \mathbb{Z}^+$  has a nonoscillatory solution.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

Consider the third-order difference equation

$$x(n+3) + 3x(n+2) - 4x(n+1) - 12x(n) = 0.$$

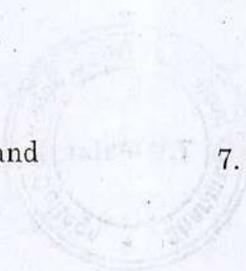
Show that the functions  $2^n, (-2)^n$ , and  $(-3)^n$  form a fundamental set of solutions of the equation.

7. Solve the system  $y(n+1) = Ay(n) + g(n)$ , where

$$A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}, g(n) = \begin{pmatrix} n \\ 1 \end{pmatrix}, y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

8. Solve the difference equation  $x(n+2) + 3x(n+1) + 2x(n) = 0, x(0) = 1, x(1) = -4$

9. State and Prove the generalization of the Poincare — Perron theorem.



10. Suppose that  $\lim_{n \rightarrow \infty} \inf p(n) = p > \frac{k^n}{(k-1)^{k+1}}$ .

Prove that the following statements hold:

- (a)  $x(n+1) - x(n) + p(n)x(n-k) \leq 0$ ,  
has no eventually positive solution
- (b)  $x(n+1) - x(n) + p(n)x(n-k) \geq 0$ ,  
has no eventually negative solution

NOVEMBER/DECEMBER 2019

MMA44 — DIFFERENCE EQUATION

Time : Three hours

Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Prove that the operator  $\Delta^{-1}$  is linear.

Or

- (b) Verify that  $\{n, 2^n\}$  is a fundamental set of solutions of the equation

$$x(n+2) - \frac{3n-2}{n-1}x(n+1) + \frac{2n}{n-1}x(n) = 0.$$

- (a) Find a general solution of the system  $x(n+1) = Ax(n)$ , where

$$A = \begin{pmatrix} 1 & -5 \\ 1 & -1 \end{pmatrix}$$

Or

- (b) Find the solution of the difference system

$$x(n+1) = Ax(n), \text{ where } A = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 2 & -4 \\ 0 & 1 & 6 \end{pmatrix}$$

