9. Solve $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\propto} \frac{\partial T}{\partial t}$

Subject to :

(a) T is bounded as $t \to \infty$

(b) $\left. \frac{\partial T}{\partial x} \right]_{x=0} = 0$ for all t

(c) $\left. \frac{\partial T}{\partial x} \right]_{x=a} = 0$ for all t

(d) $T(x, 0) = x(a - x)$, $0 < x < a$

10. Solve the one-dimensional wave equation

$u_{tt} = c^2 u_{xx}$, $0 \le x \le \pi$, $t \ge 0$

Subject to

$u = 0$, When $x = 0$ and $x = \pi$

$u_t = 0$ when $t = 0$ and

$u(x, 0) = x$, $0 < x < \pi$.

APRIL/MAY 2018

MMA23 — PARTIAL DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 6 = 30 marks)

Answer ALL questions.

1. (a) Find the integral surface of the linear partial differential equations $xp + yq = z$.

Which contains the circle defined by $x^2 + y^2 + z^2 = 4$, $x + y + z = 2$.

Or

(b) Find the characteristics of the equation $pq = z$ and determine the integral surface which passes through the straight line $x = 1$, $z = y$.

2. (a) Classify and reduce the relation $y^2 u_{xx} - 2xy u_{xy} + x^2 u_{yy} = \frac{y^2}{x} u_x + \frac{x^2}{y} u_y$ to a canonical form and solve it.

Or

(b) Reduce the following equation to a canonical form and hence solve it $u_{xx} - 2 \sin x \, u_{xy} - \cos^2 x u_{yy} - \cos x u_y = 0$.

3. (a) Explain the Neumann problem for a rectangle.

Or

(b) Solve by Separation of Variables $u_{xx} + u_{yy} = 0$.

4. (a) An infinite one-dimensional solid defined by $-\infty < x < \infty$ is maintained at zero temperature initially. There is a heat source of strength $g_s(t)$ units situated at $x = \xi$ which releases constant heat continuously for $t > 0$. Find an expression for the temperature distribution the solid for $t > 0$.

Or

(b) The ends A and B of a rod 10 cm in length, are kept at temperature 0°C and 100°C until the steady state condition prevails. Suddenly the temperature at the end A is increased to 20°C and the end B is decreased to 60°C. Find the temperature distribution in the rod at time t.

5. (a) Obtain the D'Alemberts solution of the one-dimensional wave equation.

Or

(b) Obtain the periodic solution of one dimensional wave equation in spherical polar coordinates.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

6. (a) Show that the equations $xp - yq = x$, $x^2 p + q = xz$ are compatible and hence find the solutions.

(b) Find the complete integral of $(p^2 + q^2) y = qz$ using Charpits method.

7. Verify that the Greens function for equation $\frac{\partial^2 u}{\partial x \partial y} + \frac{2}{x+y}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right) = 0$ subject to $u = 0$, $\frac{\partial u}{\partial x} = 3x^2$ on $y = x$ is given by

$$v(x, y, \xi\ \eta) = \frac{(x+y)[2xy + (\xi - \eta)(x - y) + 2\xi\eta]}{(\xi + \eta)^2}$$ and

obtain the solution of the equation in the form $u = (x - y)(2x^2 - xy + 2y^2)$

8. Find the potential at all points of space inside and outside of a sphere of radius $R = 1$ which is maintained at a constant distribution of electric potential $u(R, \theta) = f(\theta) = \cos 2\theta$.